

COMPARISON OF CIRCULAR ORBIT AND FOURIER POWER SERIES EPHEMERIS
REPRESENTATIONS FOR BACKUP USE BY THE UPPER ATMOSPHERE
RESEARCH SATELLITE ONBOARD COMPUTER

J. R. Kast, Computer Sciences Corporation

ABSTRACT

The Upper Atmosphere Research Satellite (UARS) is a three-axis stabilized Earth-pointing spacecraft in a low-Earth orbit. The UARS onboard computer (OBC) uses a Fourier Power Series (FPS) ephemeris representation that includes 42 position and 42 velocity coefficients per axis, with position residuals at 10-minute intervals. New coefficients and 32 hours of residuals are uploaded daily. This study evaluated two backup methods that permit the OBC to compute an approximate spacecraft ephemeris in the event that new ephemeris data cannot be uplinked for several days: (1) extending the use of the FPS coefficients previously uplinked and (2) switching to a simple circular orbit approximation designed and tested (but not implemented) for Landsat-D. The FPS method provides greater accuracy during the backup period and does not require additional ground operational procedures for generating and uplinking an additional ephemeris table. The tradeoff is that the high accuracy of the FPS will be degraded slightly by adopting the longer fit period necessary to obtain backup accuracy for an extended period of time. The results for UARS show that extended use of the FPS is superior to the circular orbit approximation for short-term ephemeris backup.

1.0 INTRODUCTION

The Upper Atmosphere Research Satellite (UARS) will use a Fourier Power Series (FPS) ephemeris representation (Hall and Long, 1978; Long and Folta, 1986) similar to that used by Landsat for normal onboard computation of the spacecraft ephemeris. The nominal procedure will be to uplink a new set of FPS coefficients daily along with 32 hours of residuals. Precision FPS results are required for this 32 hour timespan.

The term "ephemeris representation" is understood to include the entire process of supplying ephemeris information to the spacecraft. It includes ground-based computer generation of the spacecraft ephemeris coupled with data compression techniques, data transmission to the spacecraft, and onboard algorithms for computing the required data. Interest in a backup ephemeris representation capability for times beyond the normal 32-hour timespan led to a study in which two approaches were evaluated: (1) continuing to compute an ephemeris using the FPS coefficients without residuals, or (2) switching to a simple circular orbit approximation.

The emphasis of the study was on the fit errors introduced by substituting the FPS or circular orbit approximations as models of the predicted reference ephemeris. Although uncertainty in the predicted ephemeris itself introduces additional errors, these are the same for both FPS and circular orbits. While the effect of fit period on possible overflow of FPS coefficients and residuals (using the Landsat-D scaling parameters) was evaluated, errors introduced by overflow in onboard computer (OBC) intermediate computations were ignored. Miller (1987, p. 9) has shown that, for the FPS fits to the nominal UARS orbit used in this study, there was a sufficient cushion to avoid overflow.

The existing Landsat OBC software permits the continued use of the FPS calculation at reduced accuracy beyond the timespan of the residuals. Because the accuracy of the FPS algorithm degrades rapidly when it is evaluated at times beyond the fit period, an extended fit period for the normal FPS uplink must be considered. For this study, effects of 3-day and 7-day fits were evaluated. Lengthening the fit period extends the time period for which the FPS can be used as a backup ephemeris, but at the cost of possibly decreasing the accuracy during the normal 32-hour period of use.

The accuracy of 3-day and 7-day fits of the circular orbit approximation were evaluated and compared to FPS results. Accuracies for both techniques were computed in terms of nadir-pointing errors; along-track, cross-track, and radial position differences; velocity errors in the spacecraft body frame; and yaw, pitch, and roll errors.

The UARS OBC controls pointing of the UARS high-gain antenna (HGA) toward the Tracking and Data Relay Satellite (TDRS) by computing the required HGA gimbal angles using UARS and TDRS ephemeris results and the OBC attitude solution. Thus, extended use of the TDRS ephemeris representation would be required if UARS tracking of TDRS were to continue during the backup period. For this reason, 3-day and 7-day fits of the TDRS ephemeris representation were generated, and the Earth (nadir)-pointing errors (which can serve as a measure of the contribution of TDRS ephemeris error to the spacecraft-to-TDRS tracking error) were computed. Errors were small for either fit period.

2.0 BACKGROUND

2.1 FPS

In studies reported by Hall and Long (1978), a number of possible ephemeris representations for Earth-orbiting spacecraft with near-circular (eccentricities less than 0.02) orbits and both low-Earth (550 to 950 kilometers (km) altitude) and geosynchronous orbits were considered. Algorithms were evaluated for usefulness when computational time, data storage, and data transmission were all limited, and accuracies of 1 meter (m) to 10 km root-mean-square (rms) position error were required for timespans of up to 4 days. The FPS was selected as the spacecraft ephemeris representation to be used for onboard computation by the Solar Maximum Mission (SMM), which used the first National Aeronautics and Space Administration (NASA) Standard Spacecraft Computer (NSSC-1) OBC. The predicted ephemeris was first generated on the ground; then, using a truncated FPS, the coefficients and residuals were determined and uplinked to the OBC, where the FPS was evaluated to provide the Cartesian elements at specified time intervals.

To represent the Cartesian spacecraft ephemeris data at equispaced grid points for Landsat and for UARS, an FPS of the following form was chosen:

$$\sum_{i=1}^{N=3} \sum_{j=0}^{M=5} \left[a_{ij} t^j \sin(i\omega t) + b_{ij} t^j \cos(i\omega t) \right] + \sum_{k=0}^1 [\sin(\omega_e t) + \cos(\omega_e t)]^k [c_k \sin(2\omega_e t) + d_k \cos(2\omega_e t)]$$

where ω is the orbital frequency and is assumed to be the same for each Cartesian coordinate. The Earth's sidereal rotation frequency, ω_e , is assumed to have a value of

$$\omega_e = 2\pi/[23.934467 (3600)] \text{ radians/second}$$

In operational programs, the series is used in a nested form as follows:

$$\begin{aligned} x(t) = & A_1 + t\{A_2 + t(A_3 + t(A_4 + t(A_5 + tA_6)))\} \\ & + \{A_7 + t[A_8 + t(A_9 + t(A_{10} + t(A_{11} + tA_{12})))]\} \sin(\omega t) \\ & + \{A_{13} + t[A_{14} + t(A_{15} + t(A_{16} + t(A_{17} + tA_{18})))]\} \cos(\omega t) \\ & + \{A_{19} + t[A_{20} + t(A_{21} + t(A_{22} + tA_{23})))\} \sin^2(\omega t) \\ & + \{A_{24} + t[A_{25} + t(A_{26} + t(A_{27} + tA_{28})))\} \sin(\omega t) \cos(\omega t) \\ & + \{A_{29} + t[A_{30} + t(A_{31} + tA_{32})]\} \sin^3(\omega t) \\ & + \{A_{33} + t[A_{34} + t(A_{35} + tA_{36})]\} \sin^2(\omega t) \cos(\omega t) \\ & + \{A_{37} + A_{39} \sin(\omega t) + A_{41} \cos(\omega t)\} \sin(2\omega_e t) \\ & + \{A_{38} + A_{40} \sin(\omega t) + A_{42} \cos(\omega t)\} \cos(2\omega_e t) \end{aligned}$$

where $x(t)$ = any position or velocity component

ω = mean orbital frequency of the spacecraft for the data span

ω_e = Earth's sidereal rotation frequency

t = spacecraft clock time relative to a spacecraft clock reference time for the data span, modeled as follows (from Lee, 1981, p. 3-1; NASA, 1987, Section 3205.2.2):

$$t = t_0 + (1 + R)(T - T_0) + R_d (T - T_0)^2$$

where T = true ephemeris time

t_0, T_0 = spacecraft clock time and corresponding true time at the FPS reference time (TREF)

R = spacecraft clock drift rate

R_d = rate of change of R

For the TDRS ephemeris representation in the UARS OBC, residuals are not used and only the $A_1, A_2, A_7, A_8, A_{13}, A_{14}, A_{19}$, and A_{24} terms are fit (NASA, 1987, Section 3205.2.3).

Although some rough analogies can be made with such factors as J_2 perturbations, the individual terms of the FPS should not be thought of as having physical significance. The FPS is simply a convenient method using a limited number of terms to compute near-circular, low-Earth orbits to a suitable degree of accuracy for limited time periods.

To accurately determine the orbital frequency, ω , the maximum entropy method (MEM) is used. This method provides superior frequency resolution to Fourier analysis for short data spans; MEM can locate periodicities in the data that are of the order of the length of the data span itself without quantizing them. The MEM analysis is followed by a least squares fit of the coefficients of a truncated FPS to a precise ephemeris file generated by numerical integration. Residuals are then computed at specific grid-point times. In the OBC, these residuals can be added to the position and velocity generated by evaluating the coefficients at selected grid-time intervals to obtain Cartesian elements nearly identical to the initial precise ephemeris. A four-point Hermite interpolator is then used to obtain Cartesian elements between grid-time intervals.

2.2 CIRCULAR ORBIT APPROXIMATION

The circular orbit approximation was originally designed for use by Landsat-4 at a time when the solar panels appeared to be undergoing progressive failure (Quinn, 1984). The requirements were to provide a coarse (on the order of 1 degree) ephemeris algorithm that would need to be updated from the ground no more frequently than once per week and, if necessary, could be used for 1 month. The circular orbit algorithm uses an average nodal period, average nodal precession, and orbit radius to compute the position and velocity. For UARS, the circular orbit parameters would be fitted on the ground and uplinked as an additional OBC table each time the FPS ephemeris was uplinked.

The circular orbit model was defined by Quinn (1984, p. 4). Position and velocity are computed in GCI coordinates as follows:

$$\begin{aligned}x &= R (\cos \Omega \cos \Theta - \sin \Omega \cos i \sin \Theta) \\y &= R (\sin \Omega \cos \Theta + \cos \Omega \cos i \sin \Theta) \\z &= R (\sin i \sin \Theta) \\\dot{x} &= -V (\cos \Omega \sin \Theta + \sin \Omega \cos i \cos \Theta) \\\dot{y} &= -V (\sin \Omega \sin \Theta - \cos \Omega \cos i \cos \Theta) \\\dot{z} &= V (\sin i \cos \Theta)\end{aligned}$$

where Ω = right ascension of ascending node:

$$\Omega = \Omega (\text{at } T = 0) + T * \dot{\Omega}$$

$\dot{\Omega}$ = nodal precession rate

T = time elapsed since reference time

Θ = orbit angle (linear function of time): $\Theta = (2\pi/P)T$

P = average nodal period

R = radius at first ascending node

V = velocity at first ascending node

i = inclination of UARS orbit

2.3 EFFECT OF EPHEMERIS ERRORS ON SPACECRAFT OPERATION

It is accepted that the OBC-computed backup ephemeris may not be accurate enough for the spacecraft to meet the nadir-pointing control requirements for normal science measurements. Two coarse control requirements remain: (1) The spacecraft

line-of-sight direction to TDRS must be computed to an accuracy of 1 deg (NASA, 1987, Section 3205.2.2, p. 4) to maintain TDRS contact. (2) The spacecraft nadir-pointing error must be maintained below approximately 5 deg to avoid a transition to a safhold mode triggered by off-null Earth sensor measurements.

The UARS spacecraft will have a component of nadir-pointing error due to ephemeris representation error when it is measuring an inertial attitude and controlling to a local vertical frame computed using the onboard ephemeris. This nadir-pointing error is computed as the angle between the GCI spacecraft positions obtained from the onboard ephemeris representation and from the reference ephemeris.

When UARS is flying forward at normal attitude, the yaw axis (Z) is in the nadir direction. The pitch axis (Y) is in the direction of the negative orbit normal. The roll axis (X) is orthogonal and positive in the direction of flight. The mathematical formulation of the errors in spacecraft attitude due to errors in the OBC ephemeris representation was given by Folta (1987, Appendix A).

The nadir-pointing error is simply the root-sum-square (rss) of the pitch and roll errors, and these all depend only on position error. However, the yaw error depends on errors in the direction of the negative orbit normal, which is computed as the cross product of the spacecraft position and the velocity. Since the UARS FPS includes position but not velocity residuals, pitch and roll errors should be reduced when residuals are used, but yaw error should be relatively unaffected.

The component of spacecraft attitude error perpendicular to the spacecraft line of sight to TDRS contributes directly to error in pointing the spacecraft's high-gain antenna (HGA) at TDRS. The individual contributions from roll, pitch, and yaw depend on the geometry, but as an upper bound on error, their rss value can be used. However, the spacecraft position error also contributes to TDRS pointing error, and this error is highly correlated with pitch and roll error.

The effect of this correlation is shown in Figure 1. The normal UARS attitude control algorithm is indicated by a subscript I (applicable when the spacecraft is measuring an inertial referenced attitude and controlling to a computed nadir-pointing attitude based on the OBC ephemeris) and a possible backup control mode by a subscript E (applicable when the spacecraft is controlling to a nadir-pointing attitude based on Earth sensor measurements). UARS and TDRS actual positions are indicated by S and T; a prime (') indicates the OBC computed position. The UARS

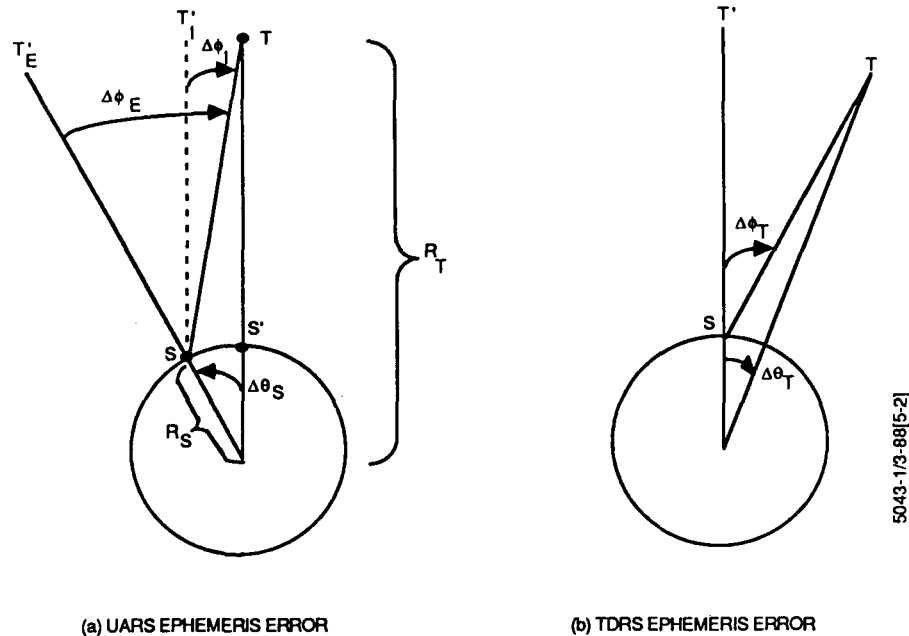


Figure 1. TDRS Pointing Error Geometry

and TDRS "computed nadir-pointing" error due to ephemeris error is indicated by $\Delta\theta_S$ and $\Delta\theta_T$.

The contribution from $\Delta\theta_S$ to spacecraft-to-TDRS pointing error is $\Delta\phi_I$ or $\Delta\phi_E$, depending on the spacecraft control mode; the contribution from $\Delta\theta_T$ is $\Delta\phi_T$. Letting S and T also represent spacecraft (6,978 km) and TDRS (42,164 km) distance from the Earth's center, it can be seen for the geometry shown that

$$\Delta\phi_I = \frac{S}{T - S} \Delta\theta_S = 0.20 \Delta\theta_S$$

$$\Delta\phi_E = \Delta\theta_S + \Delta\phi_I = 1.20 \Delta\theta_S$$

$$\Delta\phi_T = \frac{T}{T - S} \Delta\theta_T = 1.20 \Delta\theta_T$$

The equation for $\Delta\phi_I$ seems to indicate that (for normal inertial-referenced attitude control) correlations reduce the effect of ephemeris errors on TDRS pointing errors. However, when the UARS and TDRS positions are 90 deg apart, it

is clear that UARS yaw errors transform directly to TDRS pointing errors. The equation for $\Delta\theta_T$ indicates that the contribution of TDRS ephemeris errors to UARS-to-TDRSS pointing error may be approximated as the equivalent TDRS nadir-pointing error. We will therefore consider the UARS-to-TDRS pointing error to be approximated as the sum of UARS attitude error and TDRS nadir-pointing error.

3.0 PROCEDURES

Orbits generated by the FPS and the circular orbit approximation were fit and compared to a predicted ephemeris generated by the Goddard Trajectory Determination System (GTDS). The Ephemeris Representation Ground Support System (ERGSS) and a modified version of the Ephemeris Representation Ground Support Quality Assurance (ERGSQA) programs were used to compute coefficients and to generate ephemeris files past the length of time used to fit the coefficients. These programs are discussed by Boland and Lee (1982) and Boland (1982).

The circular orbit fit uses the numerical average of the nodal period over the timespan desired. The FPS fit is performed over a selected timespan of the predicted ephemeris to determine the orbital frequencies, coefficients, and residuals. Current Landsat operations use 3 days as the data span for fitting coefficients. The extended fit length used in this report was arbitrarily selected as 7 days, the largest fit length that is currently supported by the ERGSS program. Accuracies for both approximations are given for nadir-pointing angles; yaw, pitch, and roll angles; along-track, cross-track, and radial positions; and velocity components.

4.0 RESULTS

4.1 FPS AND CIRCULAR ORBIT ACCURACIES

Table 1 compares the length of time until the nadir-pointing error (defined here as the combination of pitch and roll errors) exceeds a 1-deg or 5-deg angle. These angles are as suggested by the General Electric Company for maintaining TDRS pointing and for avoiding transition to the safhold mode, respectively. As seen in the table, a 1.0-deg nadir-pointing accuracy can be maintained by either the FPS or circular orbit approximation for about the same time period if a 7-day fit

Table 1. Comparison of Length of Time From Beginning of Fit Until Nadir-Pointing Errors Exceed 1 or 5 Degrees

<u>Representation</u>	<u>Days From Beginning of Fit</u>	
	<u>1-Degree Error</u>	<u>5-Degree Error</u>
Circular		
3-day fit	9.8	21.4
7-day fit	12.7	23.9
FPS		
3-day fit	5.3	6.8
7-day fit	13.1	16.9

is used. Figures 2 and 3 present these results for 3-day and 7-day fits, respectively. Each figure shows the errors for the FPS orbit and the circular orbit when compared to a predicted ephemeris. As shown in Figure 3, from 7 days to approximately 11 days from the start of the data fit, both FPS and circular orbits give similar results for a 7-day fit. However, the FPS orbit degrades after 11 days.

Tables 2 and 3 present the position differences for both the circular orbit and FPS orbit. Maximum values are given for 3-day and 7-day fits. The significant position difference is in the along-track direction and agrees with the previous nadir-pointing result. An improvement in using the FPS instead of the circular orbit was especially noted over the first several days even after the residuals have been exhausted.

Figures 4 and 5 present the yaw errors for a 3-day fit and a 7-day fit when the FPS and circular orbit approximations are compared to a predicted ephemeris. As shown in the figures, the FPS accuracy is superior to the circular orbit accuracy during the time interval used for fitting coefficients and for a short time after for a 7-day FPS fit. Although the circular orbit has initially larger errors than the FPS orbit, it will degrade at a slower rate than the FPS orbit when significantly past the end of the time interval used for fitting coefficients. Tables 4, 5, and 6 present the yaw, pitch, roll, and velocity errors for 1-, 3-, and 7-day periods, respectively. Results represent the maximum angle or velocity errors over the stated period. During the fit period, the FPS errors are much smaller

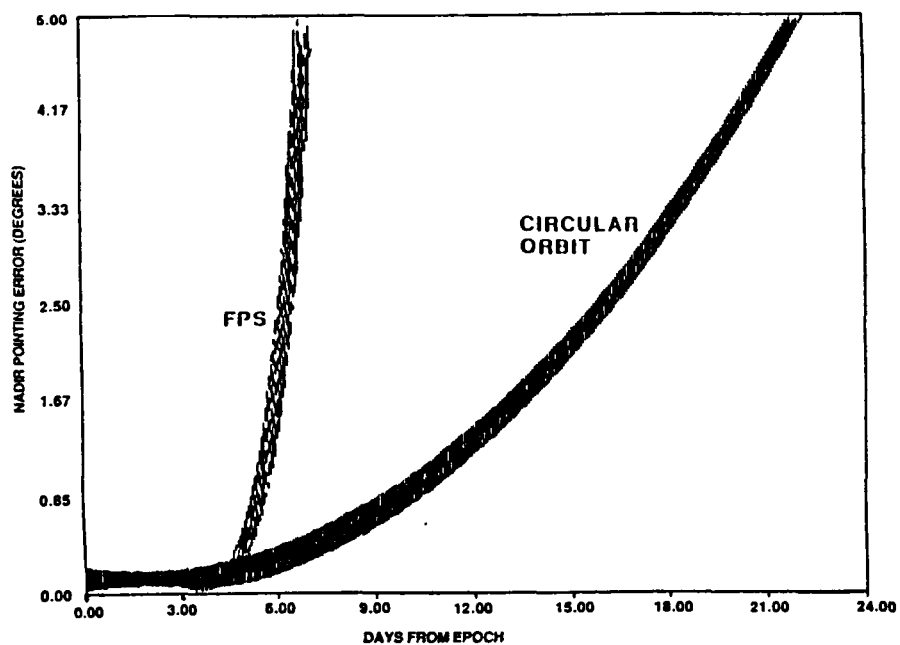


Figure 2. UARS Nadir-Pointing Error for a 3-Day Fit: FPS (No Residuals) and Circular Ephemeris Representations

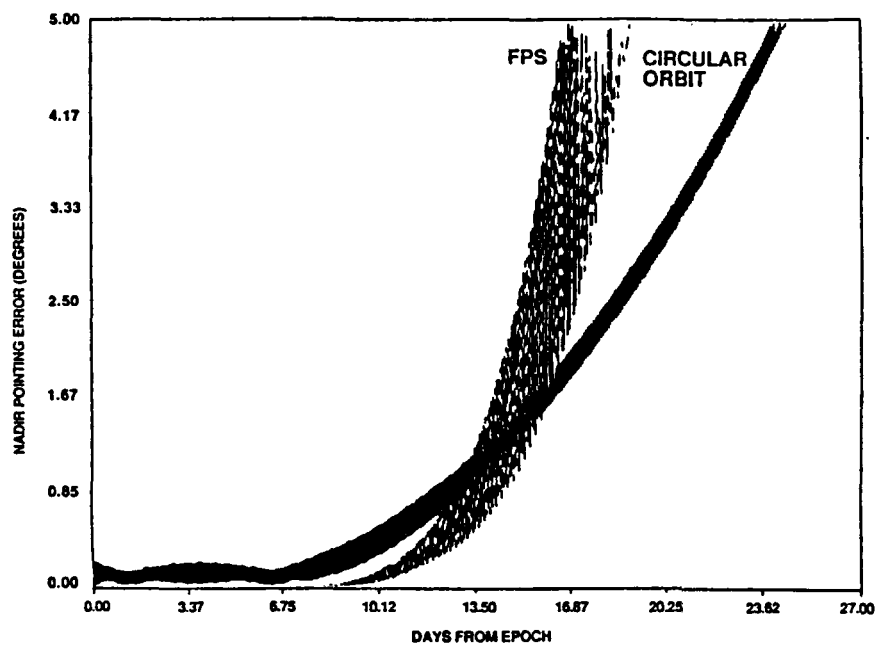


Figure 3. UARS Nadir Pointing Error for a 7-Day Fit: FPS (No Residuals) and Circular Ephemeris Representations

Table 2. Maximum Position Difference (km) During First Day of Fit

<u>Direction</u>	<u>3-Day Circular</u>	<u>7-Day Circular</u>	<u>3-Day FPS</u>		<u>7-Day FPS</u>	
			<u>No Residuals</u>	<u>With Residuals</u>	<u>No Residuals</u>	<u>With Residuals</u>
Along-track	-29.8	-29.8	0.47	0.05	0.80	0.06
Cross-track	2.7	2.9	0.13	-0.01	0.21	-0.02
Radial	11.3	11.4	0.17	-0.05	0.19	-0.05

Table 3. Maximum Position Difference (km) During First 7 Days of Fit

<u>Direction</u>	<u>3-Day Circular</u>	<u>7-Day Circular</u>	<u>3-Day FPS</u>		<u>7-Day FPS</u>	
			<u>No Residuals</u>	<u>With Residuals</u>	<u>No Residuals</u>	<u>With Residuals^a</u>
Along-track	-65.7	-29.8	795.	-	0.89	-
Cross-track	9.7	9.7	272.	-	0.27	-
Radial	11.3	11.5	146.	-	0.19	-

^aOnly 32 hours of residuals are uplinked.

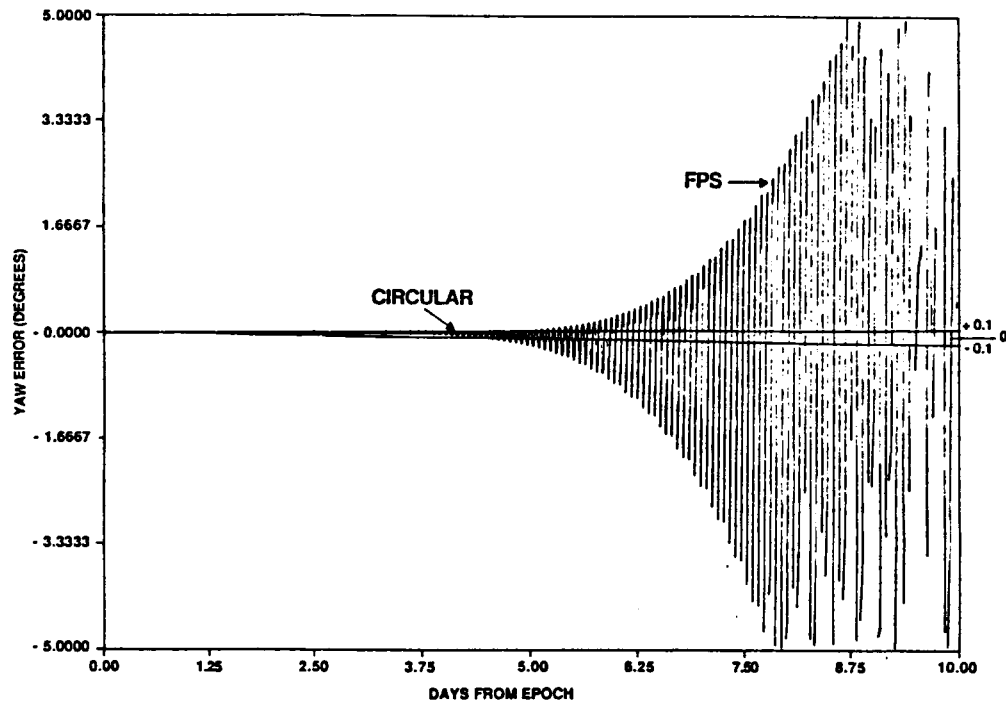


Figure 4. UARS Yaw Error for a 3-Day Fit: FPS (No Residuals) and Circular Ephemeris Representations

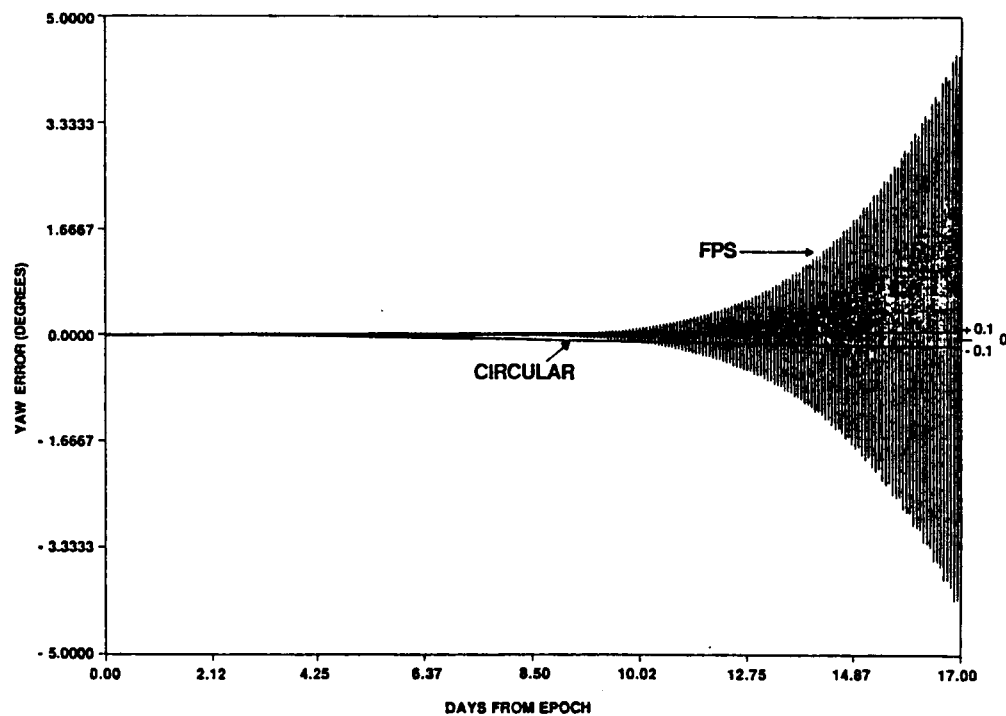


Figure 5. UARS Yaw Error for a 7-Day Fit: FPS (No Residuals) and Circular Ephemeris Representations

Table 4. Maximum Yaw, Pitch, Roll, and Velocity Errors During First Day of Fit

<u>Errors</u>	<u>3-Day Circular</u>	<u>7-Day Circular</u>	<u>3-Day FPS</u>		<u>7-Day FPS</u>	
			<u>No Residuals</u>	<u>With Residuals</u>	<u>No Residuals</u>	<u>With Residuals</u>
Pointing Errors (deg)						
Yaw	±0.0189	±0.0189	±0.0012	±0.0012	±0.0018	±0.0016
Pitch	+0.2467	+0.2451	±0.0060	±0.0003	-0.0065	±0.0005
Roll	±0.0238	±0.0232	±0.0013	±0.0001	±0.0014	±0.0001
Velocity errors (km/sec)						
X-axis	-0.0146	-0.0147		+0.0003		±0.0003
Y-axis	±0.0025	±0.0025		±0.0002		±0.0002
Z-axis	+0.0227	+0.0225		-0.0007		-0.0007
Magnitude ^a	0.0246	0.0246		0.0007		0.0008

^aMagnitude at a given epoch and not magnitude of maximum values at different epochs.

Table 5. Maximum Yaw, Pitch, Roll, and Velocity Errors During First 3 Days of Fit

<u>Errors</u>	<u>3-Day Circular</u>	<u>7-Day Circular</u>	<u>3-Day FPS</u>		<u>7-Day FPS</u>	
			<u>No Residuals</u>	<u>With Residuals</u>	<u>No Residuals</u>	<u>With Residuals</u>
Pointing Errors (deg)						
Yaw	±0.0338	±0.0338	±0.0014	±0.0014	±0.0022	±0.0022
Pitch	+0.2671	+0.2451	±0.0062	±0.0004	-0.0065	±0.0004
Roll	±0.0402	±0.0401	±0.0013	±0.0001	±0.0014	±0.0001
Velocity errors (km/sec)						
X axis	±0.0147	±0.0147		±0.0003		+0.0003
Y axis	±0.0046	±0.0036		±0.0007		±0.0008
Z axis	+0.0227	+0.0225		-0.0007		-0.0008
Magnitude ^a	0.0243	0.0268		0.0007		0.0008

^aMagnitude at a given epoch and not magnitude of maximum values at different epochs.

than the circular orbit errors. For both the circular orbit and the FPS orbit without residuals, yaw error is comparable to roll error, and pitch error is larger. The FPS pitch and roll errors are reduced by a factor of 10 by adding residuals, since computation of these angles depends only on the position. Velocity error comparisons for FPS and circular orbit again show the FPS to be superior to the circular orbit.

Table 6. Maximum Yaw, Pitch, Roll, and Velocity Errors During First 7 Days of Fit

<u>Errors</u>	<u>3-Day Circular</u>	<u>7-Day Circular</u>	<u>3-Day FPS</u>		<u>7-Day FPS</u>	
			<u>No Residuals</u>	<u>With Residuals</u>	<u>No Residuals</u>	<u>With Residuals</u>
Pointing Errors (deg)						
Yaw	±0.0802	±0.0801	-2.4100	-	±0.0022	±0.0022
Pitch	+0.5505	+0.2451	-6.4389	-	±0.0073	±0.0004
Roll	±0.0804	±0.0801	-2.3000	-	±0.0018	±0.0001
Velocity errors (km/sec)						
X axis	-0.0147	-0.0147	-0.1028		±0.0003	
Y axis	±0.0107	±0.0107	+0.3204		±0.0002	
Z axis	+0.0606	+0.0247	-0.7925		-0.0009	
Magnitude ^a	0.0638	0.0243	0.8139		0.0009	

^aMagnitude at a given epoch and not magnitude of maximum values at different epochs.

Yaw accuracy degradation over the first 3 days due to switching from a 3-day fit to a 7-day fit was analyzed. Table 7 and Figures 6 and 7 show that some slight degrading does occur. The result of extending fit lengths is to increase the yaw error from 0.0014 deg to 0.0022 deg over the first 3 days of use, which is still better than the corresponding value of 0.0338 deg for the circular orbit. Pitch and roll accuracies are not significantly affected over the first 3 days by extending the fit length, as shown in Table 5. Adding position residuals does not significantly improve the yaw accuracy.

Table 7. Results of Yaw Error Comparison During First 3 Days

<u>Representation</u>	<u>Maximum Yaw Error Observed Over First 3 Days</u>	
	<u>3-Day Fit (Degrees)</u>	<u>7-Day Fit (Degrees)</u>
FPS (with residuals)	0.0014	0.0022
FPS (without residuals)	0.0015	0.0022
Circular orbit	0.0338	0.0338

4.2 EVALUATION OF SCALING AND SIZING OF FPS COEFFICIENTS AND RESIDUALS FOR OBC USE

An analysis was performed comparing the FPS coefficients to the largest and smallest values that can be uplinked when scale factors are used to convert them for uplink to the OBC.

The UARS OBC is structured for double-precision, 36-bit double words for position and velocity coefficients and single-precision, 18-bit words for position residuals. The double-precision words do not use the sign bit of the low-order, 18-bit word; thus, only 35 bits are used. From the scale factor and number of bits used, the largest and smallest possible values of the uplink parameters can be computed from the following equations:

$$\text{Largest coefficient} = (2^N - 1)/(2^{N-S})$$

$$\text{Smallest coefficient} = 1/(2^{N-S})$$

where N = one less than the number of bits used (i.e., N = 34 for position and velocity coefficients; N = 17 for single-precision residual coefficients)

S = scale factor of OBC data taken from Landsat-D System Tables (Shirey, 1983) one for each coefficient used

These equations were used to evaluate the largest and smallest values that can be uplinked when the scale factor is taken into account. The units of the coefficients were changed from meters per (second)^P to meters per (millisec)^P, where P represents the power of time used in generating the coefficient. This was done to match the units used by the OBC. Each coefficient computed by the ERGSS program was found to be between the largest and smallest values that could be uplinked.

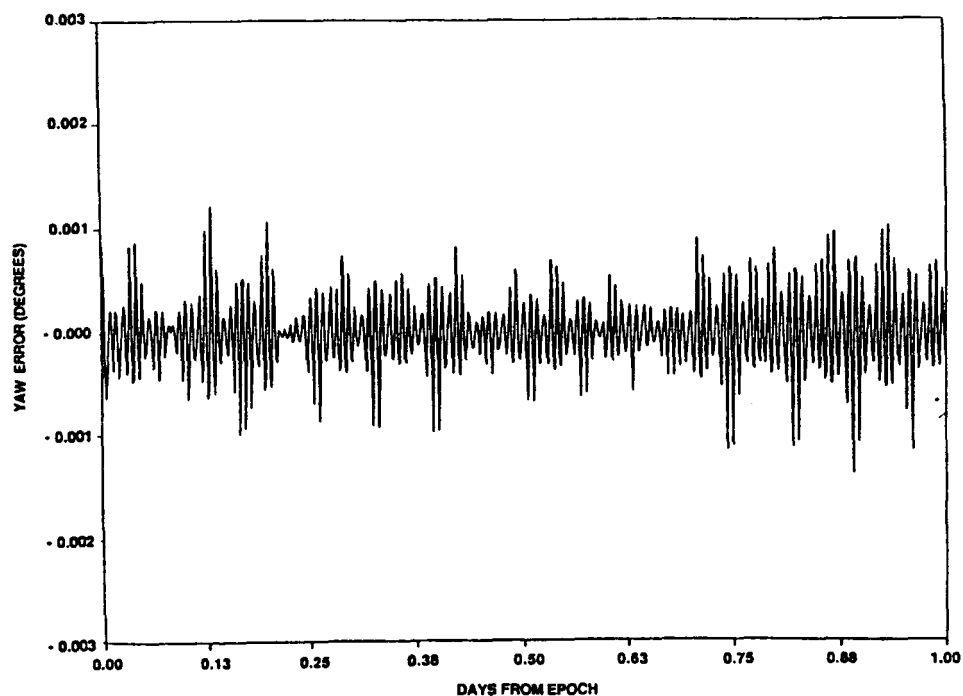


Figure 6. UARS Yaw Error During First Day for a 3-Day Fit of FPS (With Residuals)

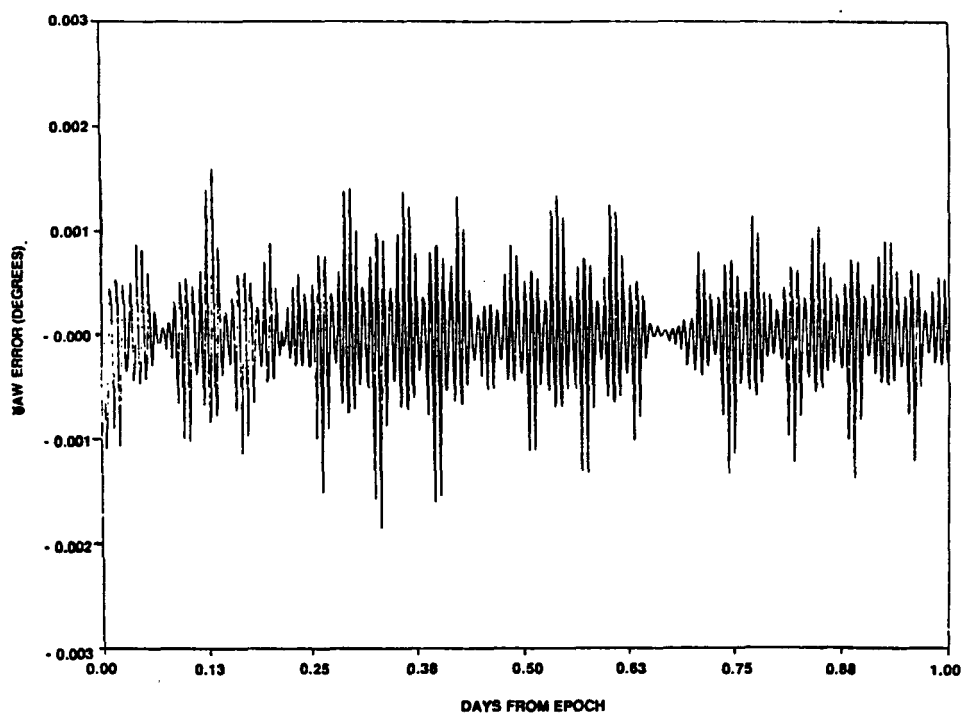


Figure 7. UARS Yaw Error During First Day for a 7-Day Fit of FPS (With Residuals)

Overflow or underflow was not encountered for the nominal eccentricity, $e = 0.001486$.

A study was next carried out (Hashmall, 1987) to determine if the Landsat scaling parameters would produce overflows using nonnominal orbital parameters. In this study, a worst-case orbit was assumed to be one with an eccentricity (e) of 0.05. Both 7-day and "standard" 3-day FPS fits were considered. The programs and procedures previously used were modified slightly to improve processing efficiency, and a search was done to determine the largest eccentricity before an overflow would occur. Most of the orbit generation runs were performed with the standard set of input orbital elements, other than eccentricity.

Additional 3-day orbit generation runs were performed for $e = 0.05$ with several values of the right ascension of the ascending node. The 172.035-deg value was reduced by 45 deg in 7.5-deg steps. Additionally, one run was done with a 90-deg decrement of the right ascension of the ascending node.

Computations of FPS coefficients were performed with a reference time (TREF) of 21 hours after the start time (as in the original study) and repeated with a TREF of 36 hours after the start time.

For the 3-day fits, overflow first occurred at $e = 0.066$, where one position coefficient, three velocity coefficients, and one residual overflowed. At $e = 0.065$, there were no overflows. For the 7-day fits, the first overflow occurred at $e = 0.049$, where a single residual overflowed. At $e = 0.048$, there were no overflows. These results were unaffected by changing the FPS reference time from 21 to 36 hours after the start of the computation interval.

FPS coefficients for 3-day fits at $e = 0.05$ showed no overflows in cases where the right ascension of the ascending node was set to values differing from the standard value by up to 45 deg.

The 3-day fit results indicated that the FPS ephemeris table scaling used for Landsat will not produce scaling problems for UARS. Even if a 7-day fit were used, an eccentricity greater than 0.048 is probably quite unlikely.

4.3 EVALUATION OF UNCERTAINTIES IN THE UARS PREDICTED ORBIT

The different scenarios for tracking passes and expected error sources, such as daily uncertainty in the solar flux or geopotential fields, result in an uncertainty in predicting the UARS orbit. Schanzle (1985, 1987) analyzed this uncertainty and reported the expected results for UARS. Figure 8 indicates a possible

total nadir-pointing error when the uncertainty in the UARS orbit is added to an FPS nadir-pointing error from a 7-day fit of coefficients (from Folta, 1987). Even though the circular orbit long-term accuracy is better than the FPS orbit accuracy, the predicted orbit uncertainty becomes the dominant error source and may exceed TDRS pointing requirements within 2 weeks.

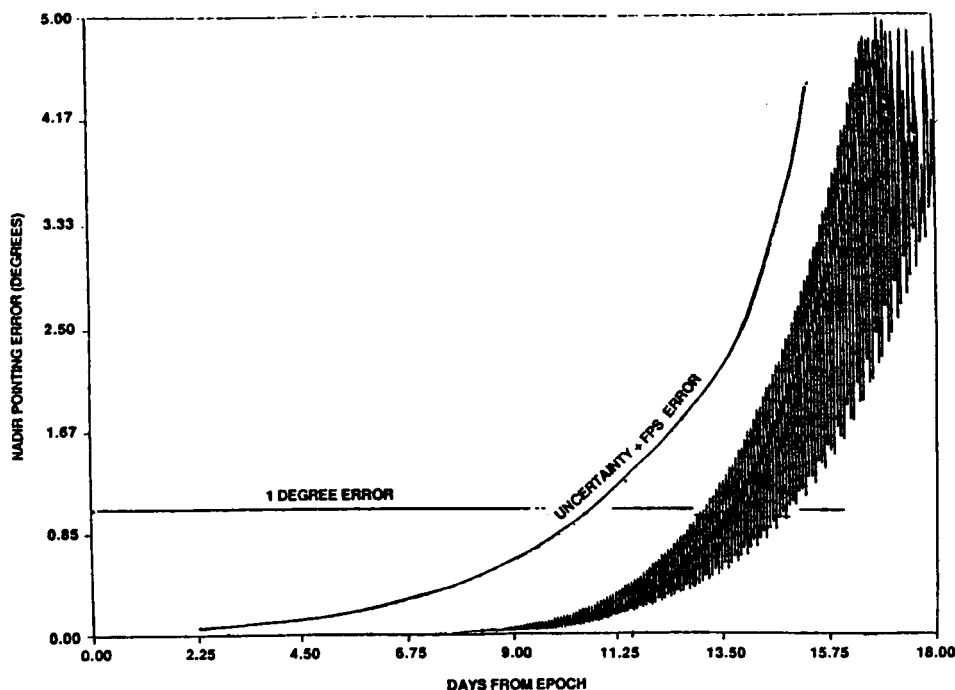


Figure 8. UARS Nadir-Pointing Error (Prediction Uncertainty Added) for a 7-Day Fit of FPS (No Residuals)

4.5 TDRS FPS ACCURACIES

UARS requires a predicted TDRS orbit to allow onboard computation of the HGA pointing angles. The TDRS orbit will be represented by FPS coefficients as described in Section 2.1, with eight coefficients uplinked to represent the position. To determine the accuracy of this representation, an analysis was performed using 3-day and 7-day FPS fit intervals. The results, presented in Figure 9, compare the TDRS nadir-pointing error to elapsed time using the same methods described in Sections 3 and 4.1. Both fit intervals yield small nadir-pointing errors over the

first 3 days, with the 7-day fit interval superior for longer periods. As indicated in Section 2.3, the contribution to spacecraft-to-TDRS pointing error, $\Delta\phi_T \approx 1.2 \Delta\theta_T$ (the computed nadir pointing error).

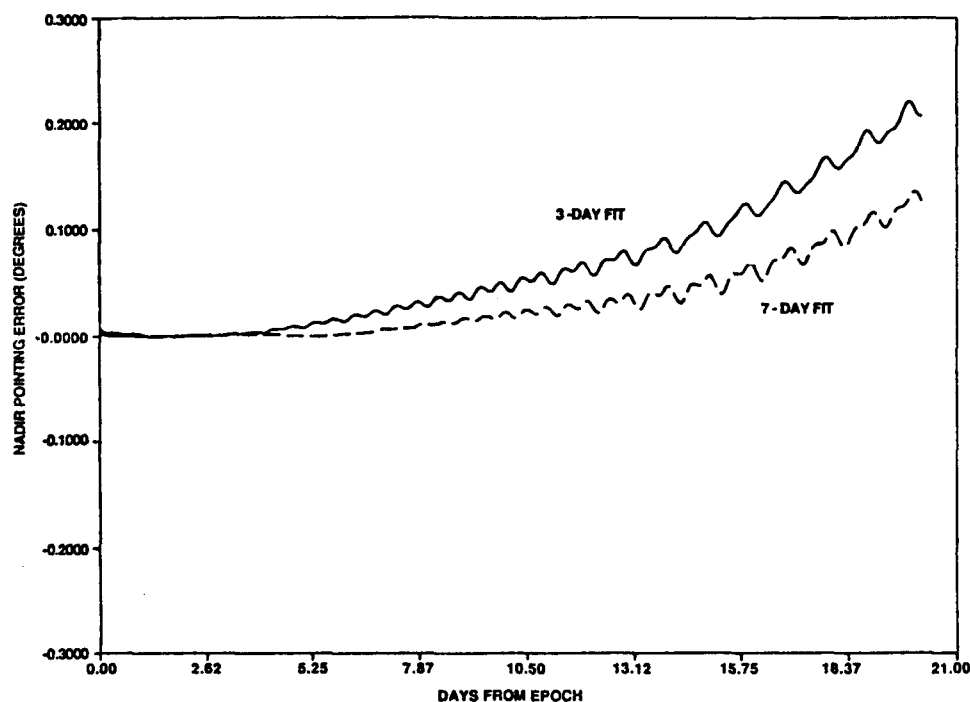


Figure 9. TDRS Nadir-Pointing Error for 3- and 7-Day Fits of FPS (No Residuals)

5.0 CONCLUSIONS

The UARS orbit can best be represented for short periods (1 week after residuals are exhausted), with coefficients generated from an FPS fit to a 7-day predicted ephemeris. Even without residuals added, the FPS orbit is superior to the circular orbit representation and should be considered for backup instead of the circular orbit technique. The results of this analysis confirmed the following:

- In general, for short-term backup (1 week), orbits generated from the FPS approximation are more accurate than those using the circular orbit approximation. Circular orbits are initially less accurate than FPS orbits but degrade more slowly over long timespans.

- A 1-deg nadir-pointing fit error and a 0.1-deg yaw fit error can be maintained by both the FPS orbit and circular orbit representations for approximately 11 days when using a 7-day fit interval.
- During the nominal uplink period, pitch and roll errors are not significantly increased by extending the fit interval from 3 days to 7 days. The maximum yaw error over the first 3 days increased from 0.0014 deg to 0.0022 deg when extending the fit length.
- Roll errors are comparable in magnitude to yaw errors for circular orbits, and for FPS orbits when position residuals are not used.
- Yaw, pitch, and roll errors, when using the FPS approximation with or without residuals, are significantly smaller than those for circular orbits over the length of the fit. When the time of comparison exceeds the timespan of the fit, the FPS accuracies degrade rapidly.
- Coefficients generated for the 7-day fit using Landsat scaling factors do not violate OBC word size requirements.
- Any increased fit accuracy in using circular orbits for long-term backup may be irrelevant because of the uncertainty in predicting the UARS orbit.
- TDRS ephemeris representation fit errors remain small (<0.1 deg) over the 1- to 2-week timespan considered in this report and do not pose a problem for spacecraft-to-TDRS pointing.

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A. Long provided background information on the ephemeris representations. Most of the work reported in this paper was performed by D. Folta. J. Hashmall extended the study of scaling and sizing of FPS coefficients to include nonnominal orbits.

APPENDIX - ELEMENTS USED FOR INPUT INTO EPHEMERIS GENERATION

The nominal osculating elements used in this analysis are as follows:

Epoch	October 1, 1989
a	6978.0653 km
e	0.0014860
i	57.017788 deg
Ω	172.03500 deg
ω	60.937802 deg
M	299.16207 deg

The propagation parameters used in this analysis are as follows:

Drag coefficient	2.2
UARS spacecraft area	0.000028 km ²
UARS spacecraft weight	5500.0 kg
Solar flux ($F_{10.7}$)	200 x 10 ²² watt/(m ² Hz)
Geopotential	15 by 15
Propagator	12th order Cowell, 60-second stepsize
Solar perturbations	Included
Lunar perturbations	Included

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